

Section 10

⑦ $f(x) = \cos x \quad f(-4\pi/3) = -\frac{1}{2}$
 $f'(x) = -\sin x \quad f'(-4\pi/3) = +\sqrt{3}/2$
 $f''(x) = -\cos x \quad f''(-4\pi/3) = +1/2$
 $f'''(x) = +\sin x \quad f'''(-4\pi/3) = -\sqrt{3}/2$

$$-\frac{1}{2} + \frac{\sqrt{3}}{2}(x+4\pi/3) + \frac{1}{2} \cdot \frac{1}{2} (x+4\pi/3)^2 - \frac{\sqrt{3}}{2} \cdot \frac{1}{3!} (x+4\pi/3)^3 + \dots$$

⑧ $f(x) = e^x \quad f(e) = e^e$
 $f'(x) = e^x \quad f'(e) = e^e$
 $f''(x) = e^x \quad f''(e) = e^e$

$$e^e + e^e(x-e) + \frac{e^e}{2!}(x-e)^2 + \frac{e^e}{3!}(x-e)^3$$

$$\sum_{n=0}^{\infty} \frac{e^e(x-e)^n}{n!} \quad \checkmark$$

⑨ $f(x) = \sin(x-2) \quad f(2) = 0$
 $x=2$
 $f'(x) = \cos(x-2) \quad f'(2) = 1$
 $f''(x) = -\sin(x-2) \quad f''(2) = 0$
 $f'''(x) = -\cos(x-2) \quad f'''(2) = -1$

$$0 + 1 \frac{(x-2)^1}{1!} + 0 \frac{(x-2)^2}{2!} + -1 \frac{(x-2)^3}{3!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{(2n+1)}}{(2n+1)!}$$

⑩ $f(x) = \arctan(x^2)$

$$\int \frac{1}{1+x^2} \approx 1 - x^2 + x^4 - x^6 + \dots \int (-1)^n x^{2n} dx$$

$$\arctan x \approx x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \dots \frac{(-1)^n x^{2n+1}}{2n+1}$$

$$\arctan(x^2) \approx \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n+1}}{2n+1}$$

$$\lim_{n \rightarrow \infty} \frac{x^{4n+6}}{2n+3} \cdot \frac{2n+1}{x^{4n+2}} = \lim_{n \rightarrow \infty} \frac{(2n+1)}{(2n+3)} (x^4)$$

$$\frac{1}{1+x} \approx \sum_{n=0}^{\infty} (-1)^n x^n \quad \text{for } x \in (-1, 1)$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2n+1}$$

$$-1 \leq x \leq 1 \quad \checkmark$$

*Check endpoints!

$$\textcircled{11} \quad f(x) = xe^x$$

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

$$\textcircled{12} \quad f(x) = 2xe^{x^2} \quad f(0) = 0$$

$$\textcircled{a} \quad f'(x) = 2e^{x^2} + 2x \cdot 2xe^{x^2} \quad f'(0) = 2$$

$$f'(x) = 2e^{x^2}(1+2x^2) \quad f''(0) = 0$$

$$f''(x) = 4xe^{x^2}(1+2x^2) + (4x)(2e^{x^2}) \quad f'''(0) = 12 \leftarrow$$

$$f''(x) = 4xe^{x^2}(1+2x^2+2)$$

$$f''(x) = 4xe^{x^2}(3+2x^2) \rightarrow \text{graph on calc to find}$$

$$\frac{12}{3!}(x-0)^3 + \frac{0}{2!}(x)^2 + \frac{2}{1}x + 0 \dots$$

$$= 2x^3 + 2x$$

$$\textcircled{b} \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots \frac{x^n}{n!} \quad \textcircled{2} \text{ yes!}$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} \dots \frac{x^{2n}}{n!}$$

$$2xe^{x^2} = 2x + \frac{2x^3}{1!} + \frac{2x^5}{2!} + \frac{2x^7}{3!} \dots$$

$$\boxed{\frac{2x^{2n+1}}{n!}}$$

✓

$$\textcircled{c} \quad \frac{d}{dx} \left[e^{x^2} = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} \dots \frac{x^{2n}}{n!} \right]$$

$$2xe^{x^2} = 2x + \frac{2x^3}{1!} + \frac{2x^5}{2!} \dots$$

$$\frac{2n}{(n-1)!} x^{2n-1} = \boxed{\frac{2x^{2n-1}}{(n-1)!}}$$

$$\textcircled{13} \quad f(x) = \cos^2 x = \frac{1}{2} + \frac{\cos 2x}{2} \quad \cos x \approx \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\frac{1}{2} + \frac{\cos 2x}{2} = \boxed{\frac{1}{2} + \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}}$$

✓

$$\begin{aligned}
 (15) \quad f(x) &= x^{1/3} & x = 8 \\
 f'(x) &= \frac{1}{3} x^{-2/3} & f(8) = 2 \\
 f''(x) &= -\frac{2}{9} x^{-5/3} & f'(8) = \frac{1}{3 \cdot 4 \cdot 2!} = \frac{1}{12} \\
 f'''(x) &= \frac{10}{27} x^{-8/3} & f''(8) = -\frac{2}{9} \cdot \frac{1}{3 \cdot 16 \cdot 2^4} = -\frac{1}{144} \cdot \frac{1}{2!} \\
 f''''(x) &= -\frac{80}{81} x^{-11/3} & f'''(8) = \frac{10}{27} \cdot \frac{1}{2^8} \cdot \frac{1}{3!} = \frac{5}{20,736} \\
 & & f''''(8) = -\frac{80}{81} \cdot \frac{10 \cdot 5}{2^4 \cdot 8} \cdot \frac{1}{4! \cdot 3 \cdot 1}
 \end{aligned}$$

$$\begin{aligned}
 y &= 2 + \frac{1}{12}(x-8) - \frac{1}{288}(x-8)^2 + \frac{5}{20,736}(x-8)^3 - \dots \\
 * \text{Graph it!} & \quad [r=8] \quad x \in (0, 16) \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 (31) \lim_{x \rightarrow 0} \frac{1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!}}{x^4} &\neq 1 = \lim_{x \rightarrow 0} -\frac{1}{2} + \left[\frac{x^4}{4!} - \frac{x^8}{6!} \dots \right] \rightarrow 0
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{\cos(x^2) - 1}{x^4} &\stackrel{H}{=} \lim_{x \rightarrow 0} \frac{-2x \sin(x^2)}{4x^3} = \lim_{x \rightarrow 0} \frac{-2 \sin(x^2)}{4x^2}
 \end{aligned}$$

$$\begin{aligned}
 &\stackrel{H}{=} \lim_{x \rightarrow 0} \frac{-4x \cos x^2}{8x} = \lim_{x \rightarrow 0} \frac{-\cos x^2}{2} = \left(-\frac{1}{2} \right) \quad \checkmark
 \end{aligned}$$

$$(33) \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - x^{12} \dots \frac{(-1)^n x^{4n}}{(2n)!}$$

$$= x - \frac{x^5}{2! \cdot 5} + \frac{x^9}{9 \cdot 4!} \dots$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!} \quad \checkmark$$

$$35) \int \frac{e^x}{x} dx = \int \frac{x^{n-1}}{n!} dx$$

$$\frac{e^x}{x} = \int \frac{1 + \sum_{k=0}^{\infty} \frac{x^k}{k!}}{x} dx = \int \left(\frac{1}{x} + \sum_{k=1}^{\infty} \frac{x^{k-1}}{k \cdot k!} \right) dx = \int \frac{x^{n-1}}{n!} dx$$

$$\int \frac{e^x}{x} dx = \boxed{\sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!} + \ln|x| + C}$$

$$39) \int_0^1 \frac{\cos(3x) - 1}{x} dx = \int_0^1 \frac{\cos(3x)}{x} dx - \int_0^1 \frac{1}{x} dx$$

$$\cos x \approx 1 - x^2 + \frac{x^4}{4!} - \frac{x^6}{6!} \dots \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos(3x) = 1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} \dots \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n}}{(2n)!}$$

$$\frac{\cos(3x) - 1}{x} = \frac{-\frac{(3x)^2}{2!} + \frac{(3x)^4}{4!}}{x} \dots \sum_{n=1}^{\infty} \frac{(-1)^n (3x)^{2n}}{x (2n)!}$$

$$\int_0^1 \frac{\cos(3x) - 1}{x} dx = \int_0^{\infty} \frac{(-1)^n (3)^{2n} x^{2n-1}}{(2n)!} = \left[\sum_{n=1}^{\infty} \frac{(-1)^n (3)^{2n} x^{2n}}{(2n)(2n)!} \right]_0^1$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (3)^{2n}}{(2n)(2n)!} \approx \boxed{S_3 = -1.575} \quad \text{Error} \leq 0.0203 \quad \checkmark$$

$$43) \text{a) } \ln(x+1) \quad \checkmark$$

$$\text{b) } \sin(2x) \quad \checkmark$$

$$\text{c) } \frac{16x^2}{2+x^2} \quad \checkmark$$

$$\text{d) } -e^{-x} + 1 \quad \checkmark$$

$$\begin{array}{l} 8x^2 - 4x^4 + 2x^6 \dots \\ \uparrow \\ \text{1st term} \\ a \end{array}$$

$$\left(\frac{-1}{2}\right)x^2 = r \quad \text{common ratio}$$

$$S_{\infty} = \frac{8x^2 \cdot 2}{1 + \frac{1}{2}x^2 \cdot 2}$$